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R&D-based Growth in the Post-modern Era[*]

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Abstract. Conventional R&D-based growth theory suggests that productivity growth is positively correlated with population size or population growth, an implication which is hard to see in the data. Here we integrate R&D-based growth into a unified growth setup with micro-founded fertility and schooling behavior. We then show how a Beckerian child quality-quantity trade-off explains why higher growth of productivity and income per capita are associated with lower population growth. The medium-run prospects for future economic growth – when fertility is going to be below replacement level in virtually all developed countries – are thus much better than predicted by conventional R&D-based growth theory.

Keywords: R&D, unified growth theory, declining population, fertility, schooling, human capital, post-modern society.

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1. Introduction

A characteristic feature of economic theories designed to explain the performance of human societies over the very long run is that they are emphasizing the interaction between economic and demographic variables as crucial for our understanding of economic development (see Galor, 2005, 2011 for surveys). Broadly speaking, these "unified growth theories" explain why the demo-economic history of countries or regions can be conceptualized as subdivided into two periods: the Malthusian era and the modern era. During the Malthusian era fertility is high, the population is gradually expanding fueled by (relatively small) productivity gains, and income is almost constant at a low level. During the modern era fertility is low and productivity gains translate into perpetual economic growth at high and (ideally) constant rates. Both eras are connected by a demographic transition during which fertility declines and the economic growth takes off. Usually it is assumed that the process ends in a state of stationary (or growing) population.

This paper introduces a third era to the analysis of long-run growth, the post-modern era. The characteristic feature of the post-modern era is a secular trend of declining population. So far, the consequences of a declining population have been relatively little researched in the field of long-run economic growth. Many theories were based on the assumption of a constant population. This assumption was until recently in line with many demographic projections, which predicted that the demographic transition comes to its end when fertility rates approach replacement level. For example, past population projections of the United Nations and the World Bank assumed in their medium variants (which were regarded as most likely) that fertility rates everywhere converge towards 2.1 births per women (Bongaarts, 1999).

Actually, however, the idea that the demographic transition stops at replacement level is refuted by empirical evidence. The total fertility rate (TFR) fell below replacement level in the 1970s in Europe and Japan, in the 1980s in North America and Australia, and in the 1990s in the Asian Tiger countries (Bongaarts, 2001). It is now below replacement level in all 50 European countries but Turkey (where it is at 2.15) and in more than 80 countries in the world (UN, 2011). Table 1.A, compiled from UN (2011), shows the most recently observed TFR for the G-8 countries, i.e. those countries that we usually associate with production at the "frontier of technological knowledge" (Aghion and Howitt, 2009). In every country that contributes substantially to innovation-based, R&D-driven growth the TFR is below replacement level.

Table 1.A: TFR for the G-8: 2005-10

USA	2.07	France	1.97
U.K.	1.83	Canada	1.65
Italy	1.38	Germany	1.36
Russia	1.44	Japan	1.32

Table 1.B: TFR USA 2008

Non-Hispanic white	1.83
Asian-American	2.05
Black	2.11
Hispanic	2.90

Among the developed countries the U.S. is unique in displaying a TFR close to replacement level. Table 1:b, compiled from U.S. National Center for Health Statistics (2010), shows that this achievement originates solely from the high TFR of the Hispanic part of the population. The TFR of non-Hispanic whites (1.83), for example, is close to that of their European forefathers. Assuming that fertility behavior of immigrants is at least partly rooted in the fertility norms of their country of origin we expect fertility of the Hispanic population in the U.S. to fall below replacement level with ongoing fertility transition in the countries of origin. Some Latin American countries (e.g. Chile, Brazil, Cuba) display already fertility below replacement and for other countries this seems to be likely in the future. In 2008 the United Nations updated their medium-variant projection, now assuming that all countries in the world converge towards a TFR of 1.85 in the long run, i.e. a fertility pronouncedly below replacement level (UN, 2008). Inspired by some recent mild recoveries of fertility the latest UN projection assumes again convergence towards replacement level, albeit with heavy undershooting; for Europe, Asia, and Latin America the TFR is predicted to remain below replacement level over the whole 21. century.

There is evidence, however, that the UN assumption of fertility rates converging towards replacement level in the medium run could be too optimistic. Strulik and Vollmer (2010) show that the countries of the world can be subdivided into two fertility groups: in one group fertility rates are converging, in the other group fertility rates are not converging, indicating that the fertility transition is not yet initiated or yet too slow for catching up with the forerunners of the transition. For the convergence-group Strulik and Vollmer show a strong linear correlation of initial fertility in 1950 (F_{50}) and fertility reduction 1950-2005 (ΔF) with no indication of leveling off at low fertility rates. The prediction implied by the estimated β-convergence equation $\Delta F = 0.82 - 0.73 F_{50}$ is a steady-state (long-run equilibrium) at a TFR of $0.82/0.73 = 1.12$, i.e. somewhat more than one child per women, almost about half of replacement fertility.

The observation that fertility is below replacement in virtually every developed country has motivated demographers to speak of “post-transitional” societies (e.g. Bongaarts, 2001). This categorization, however, could be misleading. It could be interpreted as indicating that the fertility transition has been accomplished. As shown above, this is not yet the case. Fertility rates continue to fall, although – according to β convergence – at subsequently lower rates. It may thus be more appropriate to follow van der Kaa (2001) and speak of post-modern societies.[1]

While post-modernity is a complex idea and post-modern values and their emphasis of private life and material goods (instrumental post-modernism) or the public world and social goods (humanist post-modernism) may affect virtually every aspect of life, we focus here on one aspect: the demand for children. The post-modern society is characterized by values and norms such that couples on average give birth to fewer than two children (van de Kaa, 2001, Caldwell and Schindlmayer, 2003, Preston and Hartnett, 2008). Subsequently we take preferences as given and ask for the consequences on economic growth.

According to conventional theories of R&D-based growth, the fact that the population is declining entails a grim economic outlook for post-modern societies. Models of the first generation (Romer, 1990, Aghion and Howitt, 1992) provide the result that growth of aggregate productivity (TFP) is linearly related to population size. Thus, a declining population implies vanishing growth of productivity and income per capita. According to models of the second generation (Jones, 1995, Kortum, 1997, Segerstrom, 1998), TFP growth is linearly related to population growth. If we would rule out declining productivity, these models would predict for the post-modern era stagnation of productivity and income per capita.[2]

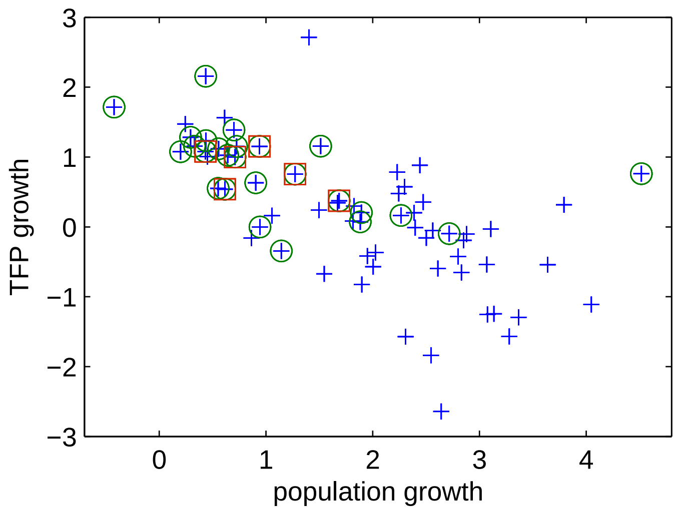
Fortunately, the empirical evidence does not support these predictions. Many studies have demonstrated a negative association between population growth and income growth (e.g. Brander and Dowrik, 1994, Kelley and Schmidt, 1995, Ahituv, 2001, and Herzer et al., 2010). Also the positive association between population growth and productivity growth predicted by conventional R&D-based growth theory is hard to see in the data. Because knowledge spillovers decline with distance and are smaller across countries than within countries (Jaffe et al., 1993, Keller, 2002, Bottazzi and Peri, 2003), we would expect that at least some of the high TFP growth generated in countries where population growth is high to be visible in the data. Figure 1 shows average annual population growth against average annual TFP growth from 1950 to 2000 (calculated from the data in Baier et

[1] In the very long run it is probably also hard to imagine that world population declines forever, i.e. until extinction. At some point we may expect that economic mechanism increase the rewards for children strongly enough to initiate a turn of the fertility transition towards convergence to replacement level from below.

[2] R&D Models of the third generation (Peretto, 1998; Young, 1998; and Howitt, 1999) combine features of the earlier generations by investigating quality R&D and variety R&D. Assuming that there exist no knowledge spillovers between quality and variety R&D they predict that only variety growth is essentially associated with population growth while constant quality growth requires a constant population. See Jones (1999) for a survey.

al, 2006). Across all countries for which data is available (identified in the Figure by blue crosses) the simple correlation is clearly negative (see Bernanke and Guerkaynak, 2001, for a similar finding).

Figure 1: Population Growth vs. TFP Growth 1950 - 2000



Growth rates are average annual growth rates 1950-2000 calculated from Baier et al. (2006). Blue crosses: all available countries, green circles: OECD countries, red squares: G7 countries.

For a proper check of R&D-based growth theory, however, it seems reasonable to reduce the sample, acknowledging the fact that less developed countries – where usually population growth is highest – do not much advance TFP growth by market R&D activities. But if we focus just on OECD countries (green circles in the Figure) the predicted positive association is still not visible. Even if we assume that conventional R&D-based growth theory applies foremost to the G7 countries, i.e. a small group of countries that pushes the world technology frontier (identified in the Figure by red squares), the predicted positive association remains invisible.[3]

In order to explore the association between TFP and population growth a bit further, we constructed TFP growth rates and growth rates of the labor force in ten year steps between 1940 and 2000 for a sample of 67 countries using the data from Baier et al. (2006). This allows us to expand the sample size considerably and, more importantly, it enables us to control for country- and time-specific fixed effects. The model that we estimate reads

$$g_{i,t} = \beta_1 + \beta_2 \Delta \log(L_{i,t}) + \epsilon_i + \kappa_t + u_{i,t}, \tag{1}$$

[3]Data for Germany is missing in Figure 1 and the regressions below. But given (West-) Germany's exceptionally low fertility rates and relatively high TFP growth rates, it can be conjectured that including Germany would certainly corroborate the result.

where $g_{i,t}$ is average TFP growth in country i between time t and $t-1$, $\Delta\log(L_{i,t})$ is the average growth rate of the labor force in county i between time t and time $t-1$, ϵ_i are country specific fixed effects, κ_t are time specific fixed effects, $u_{i,t}$ is the error term and β_1 and β_2 are the coefficients to be estimated. The results are reported in Table 2 for the total sample referred to as "World", the OECD countries referred to as "OECD" and for the G7 countries except Germany.

Table 2: The Association between TFP and Population Growth β_2

	two-way fe			two-way re			cross-section fe			pooled OLS		
	World	OECD	G7	World	OECD	G7	World	OECD	G7	World	OECD	G7
β_2	-0.89	-1.11	-1.04	-1.10	-1.04	-0.77	-1.08	-1.33	-0.95	-1.21	-1.16	-0.77
t-value	-5.29	-3.79	-0.89	-8.48	-4.83	-1.05	-6.18	-4.38	-0.93	-8.61	-4.79	-0.98
R^2	0.36	0.33	0.17	0.15	0.13	0.03	0.25	0.19	0.05	0.17	0.14	0.03
N	402	156	36	402	156	36	402	156	36	402	156	36

List of countries: Algeria, Argentina, Australia, Austria, Belgium, Bolivia, Brazil, Bulgaria, Canada, Chile, China, Colombia, Costa Rica, Cyprus, Denmark, Dominican Republic, Ecuador, Egypt, El Salvador, Ethiopia, Finland, France, Greece, Guatemala, Guyana, Honduras, Hungary, India, Indonesia, Iran, Iraq, Ireland, Israel, Italy, Jamaica, Japan, Mexico, Morocco, Netherlands, New Zealand, Nicaragua, Nigeria, Norway, Pakistan, Panama, Paraguay, Philippines, Poland, Portugal, Romania, South Africa, South Korea, Spain, Sri Lanka, Sweden, Switzerland, Syria, Thailand, Tunisia, Turkey, UK, Uruguay, USA, Venezuela, Zaire, Zambia, Zimbabwe.

As Table 2 documents, the central qualitative result of a negative correlation between TFP growth and population growth is robust to model specifications with respect to country- and time-specific fixed effects. Furthermore, considering a random effects estimator does not change the qualitative results either. The correlation is highly significantly negative for the world and the OECD. For the G7, which is actually only a "G6" since data for Germany is missing, the estimate loses precision. Given that there are only six countries, including one prominent outlier (the US), this seems to be a natural result. In any case, we do not find supporting evidence for a positive correlation between TFP growth and population growth. In our favorite specification, the two-way fixed effects regression, the estimated slope is about unity across samples, suggesting that a decrease of population growth translates one-to-one into an increase of TFP growth. Given the possibility of reverse causality, these exercises are, of course, not sufficient to reject a potential causal positive impact of population growth on TFP growth. But it is hard to come up with a mechanism strong enough to overturn such a positive impact so that it becomes invisible in the data.

Shifting the focus towards population levels and a historical perspective of technology evolution over the very long-run, Comin et al. (2010) have recently shown that across countries the current level of technology is positively associated with the level of technology in the year 1500 and *negatively* associated with population size in 1500. Interestingly they have also shown a positive association of population size in year 0 AD with the level of technology in 1500. Thus the population push view that a larger population produces more ideas (see also Kremer, 1993, Strulik and Weisdorf, 2008) seems to be true in the ancient and medieval past but not for modern societies. The present paper offers an explanation for this phenomenon by arguing that the reversal occurred when a child-quantity substitution became operative and parents began to invest in education of their children.

Below we propose a refined view on the human factor in TFP growth and argue that it is not the sheer number of workers (L) that propels the creation of ideas and the advancement of productivity but the total amount of knowledge embodied in these workers, i.e. aggregate human capital (H). The most intuitive aggregation is probably that aggregate human capital is given by human capital per worker h times the number of workers ($H = h \cdot L$). Utilizing this notion of human capital and endogenizing the incentive to acquire it through costly schooling, a couple of papers have demonstrated that human capital growth can take over the role of population growth in R&D-based growth models by predicting that productivity growth can be sustained with constant or declining population as long as human capital is accumulated rapidly enough. This prediction is less easily refuted by the data since empirical evidence supports a positive association between proxies of human capital accumulation and growth of income per capita and TFP.[4]

While the integration of human capital accumulation into R&D-based growth theory provides a way around the need for constant population size or a positive rate of population growth in order to sustain long-run economic growth, the so far available literature has left unsolved the problem of the potentially negative association of population growth and TFP growth. To be specific, acknowledging that aggregate human capital, $H = h \cdot L$, matters for the creation of new ideas, the fundamental problem is to explain why productivity growth seems to be positively associated with increasing h and negatively associated with increasing L. This problem remained unsolved because the available literature has neglected the *interaction* of quantity and quality of the workforce.

Indeed, there exists no obvious way to explain at the *macro*-level how L and h could potentially contribute conversely to the aggregate $h \cdot L$. On the micro-level, however, there exists a well-established and tested theory precisely for this, the Beckerian child quantity-quality trade-off (Becker, 1960, Rosenzweig, Wolpin, 1980, Rosenzweig, 1990, Hanushek, 1992, Becker et al., 2010, Lee and

[4]For theory see, among others, Arnold (1998), Funke and Strulik (2000), Dalgaard and Kreiner (2001), Strulik (2005), Dalgaard and Jensen (2009), Grossmann (2010). For evidence see Bernanke and Guerkaynak (2001), Barro (2001), Krueger and Lindahl (2001). Authors of the original R&D-based growth model sometimes acknowledge the fact that it is H rather than L that drives the development of new ideas, see e.g. Romer (1990). However, this observation has not motivated them to integrate an explanation of the accumulation of H into the model.

Mason, 2010). This mechanism, which plays also a crucial role in unified growth theory (Galor, 2005), allows parents to substitute child quality for child quantity such that h rises and L falls. If the substitution is such that h rises more strongly than L falls, the micro-foundation can motivate that aggregate human capital H in a society rises although the population declines. If, in turn, the development of ideas and thus TFP growth is driven by H, the micro-foundation explains why we observe a negative association between TFP growth and population growth at the macro-level.

Utilizing these ideas, the present paper integrates for the first time R&D-based growth into a unified growth theory based on a micro-founded child quantity-quality trade-off and shows why and how the preference for less children promotes human capital accumulation and economic growth. This way, R&D-based growth theory is accommodated to the evidence on education, fertility and TFP growth. At the same time the "old" theory is not completely abandoned. It is still there when the corner solution for education applies. If preferences do not support a quantity-quality substitution, increasing fertility and population growth contribute positively to economic growth as evidenced for most of human history. On the other hand, if the quantity-quality trade-off is operative, the direction of the aggregate effect is independent from family size and, in particular, also observed for fertility below replacement level. Taken together these results identify child quantity-quality substitution as the causal driver of R&D-based growth for post-modern societies.[5]

The paper is organized as follows. The next section sets up the model. Section 3 analyzes the balanced growth path and proves our main results. Section 4 specifies the model numerically and investigates adjustment over the the very long-run. It demonstrates that our theoretical results for the balanced growth path hold also true along the transition. Furthermore it offers novel insights about the onset of innovation based growth (the first Industrial Revolution) and mass education driven growth (the second Industrial Revolution). The final section concludes with a tentative outlook for future economic development.

2. The Model

2.1. **Households.** Consider an economy populated by three overlapping generations, children, young adults, and old adults. Children consume the provisions received by their parents and old adults consume their savings plus interest. Young adults supply one unit of labor and decide how to split

[5]So far, a few articles have integrated endogenous fertility into R&D-based growth, notably Jones (2001), Connolly and Peretto (2003) and Growiec (2006). Articles integrating education have been referenced above. To our best knowledge, an integration of R&D-based growth theory into unified growth theory, that is into a framework in which both fertility and education are micro-founded, does not yet exist.

their income between current consumption and future consumption, how many children they want to have, and how much they want to spend on their children's education.

In order to convey the basic theory conveniently and to get explicit solutions, we make a number of simplifying assumptions. Each household consists of one parent (which avoids to tackle matching problems), there is no explicit consideration of mortality (which avoids problems of uncertain survival), children are a continuous number (which avoids problems of indivisibility), and the motive of child expenditure is non-operational (which avoids problems of maximizing dynastic value functions). This means that parents' motivation to spend on children's education is not driven by the anticipation of the increase of children's utility caused by this expenditure but by a "warm glow" of giving (Andreoni, 1989) or the desire for having "higher quality"children (Becker, 1960).

To be specific let c_t^1 and c_t^2 denote consumption of the young and old in period t. The currently young, facing a gross interest rate R_{t+1}, and making a savings decision s_t, expect future consumption $c_{t+1}^2 = R_{t+1}s_t$. A young adult's human capital is denoted by h_t and the wage per unit of human capital is denoted by w_t. Let n_t denote the number of children and τ the time cost involved in having a child.[6] Children acquire a minimum skill-level of $\bar{e}$ by observing and imitating parents and peers at work. To increase skills beyond this minimum level parents may spend e_t per child, conceptualized in the Beckerian sense as child quality expenditure. Plugged into a function for education, education determines the human capital endowment of next period's generation (h_{t+1}). Since the parameters of education and future wages are given to the single adult, having expenditure on education $\bar{e} + e_t$ or next period's endowments h_{t+1}, or wage income of their children $w_{t+1}h_{t+1}$ in the utility function leads to equivalent results. Summarizing, young adults solve the problem

$$\max_{c_t, s_t, e_t, n_t} u_t = \log c_t^1 + \beta \log(R_{t+1}s_t) + \gamma \log(\bar{e} + e_t) + \eta \log n_t$$

subject to the budget constraint $w_t h_t(1 - \tau n_t) = c_t^1 + s_t + n_t e_t$. All variables have to be non-negative. The positive parameters β, γ, and η denote the weights of future consumption, child expenditure, and family size for utility, i.e. the importance of these elements relative to current consumption. In order to get a meaningful problem in which a population of positive size exists, we assume $\eta > \gamma$, which ensures that $n_t > 0$. With respect to education, however, no such logically argument can be made, implying that e_t could be positive or zero depending on whether the non-negativity constraint $e_t \geq 0$ is binding or not.

[6]Following Galor (2005) n_t could be interpreted as the number of children up to adulthood, implicitly assuming that child costs are only associated with surviving children.

From the first order conditions we obtain the solution (2) for consumption and savings regardless of whether education is interior or at the corner.

$$c_t = \frac{1}{1+\beta+\eta} \cdot w_t h_t, \qquad s_t = \frac{\beta}{1+\beta+\eta} \cdot w_t h_t. \tag{2}$$

For child quantity and quality there exists a threshold at $z \equiv \eta\bar{e}/(\gamma\tau)$. If income falls below the threshold parents do not invest in education and focus on maximizing child quantity. In particular we obtain from the first order conditions

$$e_t = \begin{cases} 0 & \text{for } w_t h_t < z \\ \dfrac{\gamma\tau w_t h_t - \eta\bar{e}}{\eta-\gamma} & \text{otherwise} \end{cases} \tag{3}$$

$$n_t = \begin{cases} \dfrac{\eta}{(1+\beta+\eta)\tau} & \text{for } w_t h_t < z \\ \dfrac{(\eta-\gamma)w_t h_t}{(1+\beta+\eta)(\tau w_t h_t - \bar{e})} & \text{otherwise .} \end{cases} \tag{4}$$

Once income surpasses the threshold z a fertility transition is initiated: further rising income leads to declining fertility and increasing expenditure for education. While education expenditure is not bounded, fertility arrives at a lower bound as income approaches infinity.

$$\lim_{w_t h_t \to \infty} n_t = \underline{\mathrm{n}} \equiv \frac{\eta-\gamma}{(1+\beta+\eta)\tau}. \tag{5}$$

Note that nothing prevents $\underline{\mathrm{n}}$ to fall below the replacement rate of unity. In particular, $\underline{\mathrm{n}} < 1$ for $\tau > (\eta-\gamma)/(1+\beta+\eta)$. While most of the (unified) growth literature implicitly or explicitly assumes that the fertility transition ends at a fertility rate of unity, this paper focusses on the case where preferences for child quantity η are sufficiently low, or preferences for quality γ or time costs of children τ are sufficiently high such that the fertility transition eventually drives n_t below unity. We associate this phenomenon – in line with the arguments developed in the Introduction – with a post-modern society.

2.2. **Education.** Child expenditure e_t is transformed into human capital of the next generation of young adults via a schooling technology. A reasonable technology does not just translate expenditure one to one into human capital but controls also for the costs of schooling. These costs can be conveniently approximated by the wage w_t, i.e. the cost of a unit of human capital of the current adult (teacher-) generation. The simplest conceivable schooling technology is given by $h_{t+1} = A_E(e_t/w_t)+\bar{e}$ in which A_E signifies general productivity of schooling. Without education expenditure, human capital of the next generation consists of basic skills picked up from observing and mimicking parents and peers. Inserting (3) into the schooling technology provides a simple equation of motion for human capital:[7]

$$h_{t+1} = A_E \left(\frac{\gamma\tau}{\eta - \gamma} h_t - \frac{\eta\bar{e}}{w_t} \right) + \bar{e}. \tag{6}$$

With growing income $w_t h_t$ human capital creation converges towards a linear difference equation, that is the gross growth rate becomes a constant.

$$\lim_{w_t h_t \to \infty} \frac{h_{t+1}}{h_t} = \Delta_h \equiv A_E \frac{\gamma\tau}{\eta - \gamma}. \tag{7}$$

The fact that Δ_h is constant permits perpetual long-run growth (if Δ_h is larger than unity). We discuss the case of impossible perpetual growth in the Conclusion.

2.3. **Firms: Overview.** The setup of firms and markets follows closely Romer (1990) and Jones (1995). The economy consists of three sectors: The R&D-sector is perfectly competitive and employs scientists to create new ideas in the form of blueprints, manifested in patents. A patent is needed as fixed input in a monopolistically competitive sector to produce a specialized capital good. Purchase of a patent allows a capital goods producer to transform one unit of raw capital, i.e. one unit of individual's savings, into one blueprint-specific machine. A perfectly competitive final goods sector uses these machines and workers to assemble a consumption aggregate.

Aside from the setup in discrete time the "only" modification of the firm's side of the Romer-Jones model is that the human factor in production is human capital $H_t = h_t L_t$ where L_t is the size of the current generation of young adults. Note that this aggregation of individual human capital h_t implies an infinite elasticity of substitution between human capital per person and persons. It means that any lack of human capital that a firm's currently employed workers may display can be taken care of by just employing more workers of the same skill level.

2.4. **Final goods sector.** Since the firms' side of the model – aside from the special role of human capital – coincides with the Romer-Jones setup, description can be brief. The final goods sector

[7]Whereas some elements of the schooling function could be made more general, controlling for the teacher- generation's wage is essential for stability. Otherwise human capital would grow hyper-exponentially, driven by increasing h_t *and* rising w_t. A similar control for the current state of quality is known to be essential for stability in R&D-driven quality improvements of products, see e.g. Li (2000).

operates a Cobb-Douglas production technology

$$Y_t = B_t (H_t^Y)^{1-\alpha} \sum_{i=1}^{A_t} x_{i,t}^{\alpha} \tag{8}$$

in which Y_t is output, B_t is factor productivity, and H_t^Y is employment. The parameter α is the capital share in final goods production, $x_{i,t}$ is the amount of a certain machine i used in final goods production and A_t is the number of available differentiated inputs. Facing a wage w_t per unit of human capital, and rental prices $p_{i,t}$ for capital inputs $i = 1, \ldots, A$, the indirect demand functions are given by

$$w_t = (1-\alpha) B_t (H_t^Y)^{-\alpha} \sum_{i=1}^{A_t} x_{i,t}^{\alpha} = (1-\alpha) \frac{Y_t}{H_t^Y} \tag{9}$$

$$p_{i,t} = \alpha B_t (H_t^Y)^{1-\alpha} (x_{i,t})^{\alpha-1}. \tag{10}$$

2.5. **Capital Goods Production.** Producers of specialized inputs transform one unit of raw capital into one unit of specialized capital such that $k_t = x_t$. Operating profits of an intermediate goods producer $\pi_{i,t}$ are thus given by $\pi_{i,t} = p_{i,t}(x_{i,t}) k_{i,t} - r_t k_{i,t} = \alpha B_t (H_t^Y)^{1-\alpha} (k_{i,t})^{\alpha} - r_t k_{i,t}$ where r_t denotes the interest rate that has to be paid for individual's savings. Solving the associated problem of profit maximization facing demand (10) leads to the price of $p_{i,t} = p_t = r_t/\alpha$ for all $i = 1, \ldots, A$ types of machines so that the machine-specific index can be dropped.

Free entry into capital goods production implies that in equilibrium operating profits are covering the fixed costs of production originating from purchasing a patent. In slight deviation from the original setup and inspired by Aghion and Howitt (2009, Chapter 4) we assume that a patent holds for one period (i.e. one generation) and that afterwards the monopoly right to produce a good passes to someone chosen at random from the next generation. This simplification helps to avoid intertemporal (dynastic) problems of patent holding and patent pricing while keeping the basic incentive to create new knowledge intact. Summarizing, free entry implies $\pi_{i,t} = \pi_t = p_t^A$ where p_t^A is the price of a patent (blueprint).

Because capital goods are sold at the same price and demanded at equal quantities, $x_{i,t} = x_t$ and they can be added up to the aggregate capital stock $K_t = A_t x_t$. Inserting this information into the production of final goods, equation (8) simplifies to

$$Y_t = B_t A_t^{1-\alpha} (H_t^Y)^{1-\alpha} K_t^{\alpha}. \tag{11}$$

The number of developed specialized inputs appears as aggregate productivity in goods production. We thus have two forces driving aggregate productivity. First, A_t which is driven by the development of new products through market R&D and, second, B_t which may rise for exogenous reasons, most naturally through learning by-doing.

Standard R&D-based growth theory usually neglects learning-by-doing because it focusses solely on modern economies. Here, within a unified growth setting, which encompasses centuries or millennia of economic development, we need a force to drive productivity growth before the onset of market R&D. In line with Kremer (1993), Galor and Weil (2000), Galor (2005) we assume that these learning-by-doing activities depend positively on the scale of the economy measured by population size, $(B_{t+1} - B_t)/B_t = \tilde{g}(L_t)$, $\partial \tilde{g}/\partial L \geq 0$, $\partial^2 \tilde{g}/\partial L^2 \leq 0$. The learning-by-doing mechanism is appropriate to investigate technological and economic development for most of human history because technological advances were not (much) brought forward by formally trained scientists before the industrial revolution (Mokyr, 2002). Thus, while integrating learning-by-doing activities is unnecessary with respect to our main objective, that is the analysis of R&D-driven growth in the post modern society, it adds – at little notational cost – more realism to the predicted historical adjustment dynamics for the very long-run.

2.6. **R&D.** Between periods t and $t+1$ competitive R&D-firms employ H_t^A researchers to develop $A_{t+1} - A_t$ new blueprints and sell them at price p_t^A. Facing research productivity δ output is given by

$$A_{t+1} - A_t = \delta H_t^A. \tag{12}$$

Research productivity δ is given to the single firm but depends, on the aggregate level, positively on the number of already existing ideas ($0 < \phi < 1$, standing-on-shoulders effect) and possibly negatively on the size of the workforce ($0 \leq \nu < 1$, stepping-on-toes), $\delta = \bar{\delta} A^{\phi} L^{-\nu}$, where $\bar{\delta} > 0$ is a scaling parameter. Note that the negative stepping-on-toes effect increases in physical labor L_t not in aggregate human capital H_t. The reason is that there cannot be stepping-on-toes with respect to individual h_t because the same person will not develop the same idea twice.

Maximization of profits $p_t^A \delta H_t^A - w_t H_t^A$ leads to labor demand such that $w_t = \delta p_t^A$. Labor demand in research adds up with labor demand in final goods production to aggregate labor demand

$$H_t = H_t^A + H_t^Y. \tag{13}$$

2.7. **Market Clearing and Equilibrium Dynamics.** In equilibrium, wages in goods production and R&D equalize such that $\delta p_t^A = (1-\alpha)Y_t/H_t^Y$. By inserting demand (10) into the goods price $p_t = r_t/\alpha$ and the result into profits, the free entry condition can be written as $p_t^A = \pi_t = \alpha(1-\alpha)Y_t/A_t$. Next, use these two equations for p_t^A to eliminate the price of blueprints and to arrive at labor demand $H_t^Y = A/(\alpha\delta)$ and thus $H_t^A = H_t - A/(\alpha\delta)$.

Inserting employment of researchers H_t^A from (13), the definition of R&D productivity δ, and the definition of aggregate human capital into research output (12) provides the evolution of TFP as a function of current TFP A_t, human capital per person h_t, and size of the workforce L_t,

$$A_{t+1} = \bar{\delta} A_t^{\phi} h_t L_t^{1-\nu} - \frac{1-\alpha}{\alpha} A_t, \tag{14}$$

which constitutes the human-capital augmented Romer-Jones result. In contrast to standard R&D-based growth theory we want to include also an era without market R&D in order to explore the model in a unified growth framework. There is no R&D when the non-negativity constraint for employment in R&D, $H_t^A \geq 0$ is binding, that is for $H_t \leq A/(\alpha\delta)$. If the aggregate stock of human capital is sufficiently low all labor supply is absorbed by the final goods sector and R&D as a market activity does not take place.

The population grows at the fertility rate, implying that next period's workforce is

$$L_{t+1} = n_t L_t. \tag{15}$$

For convenience physical capital is assumed to fully depreciate between periods t and $t+1$ such that next period's capital stock consists of this period's savings. Inserting into $K_{t+1} = s_t L_t$, the solution for savings (2) and wages from (9) and (11) and substituting $H_t^Y = A/(\alpha\delta)$ provides evolution of aggregate capital as (16).

$$K_{t+1} = \tilde{B}_t K_t^{\alpha} A_t^{1-\alpha-\alpha(1-\phi)} h_t L_t, \tag{16}$$

with $\tilde{B}_t \equiv \beta(1-\alpha)(\alpha\bar{\delta})^{\alpha}/(1+\beta+\eta)B_t$. The evolution of the economy is fully described by the four-dimensional system (6) and (14)-(16).

3. The Balanced Growth Path

3.1. **The Inverse Correlation between Productivity Growth and Population Growth.** A balanced growth path (BGP) is defined as a state of the economy at which growth rates do not change. For any variable x, the growth rate is denoted by $g_{x,t} = (x_{t+1} - x_t)/x_t$ and its rate of change by $\hat{g}_{x,t} \equiv (g_{x_{t+1}} - g_{x_t})/g_{x_t}$. Balanced growth thus requires $\hat{g}_x = 0$ for $x = A, K, h, L$. We denote a growth rate of x along the BGP by g_x, i.e. by omitting the time index. Naturally, because of decreasing returns of learning-by-doing, $g_B = 0$. Along the BGP productivity growth is solely driven by market R&D. For $\hat{g}_A = 0$ we obtain from (14) that along the BGP

$$\left(\frac{A_{t+1}}{A_t}\right)^{1-\phi} = \left(\frac{h_{t+1}}{h_t}\right)\left(\frac{L_{t+1}}{L_t}\right)^{1-\nu} = \left(\frac{h_{t+1}}{h_t}\right) n_t^{1-\nu}. \tag{17}$$

Superficial inspection thus *seemingly* suggests that TFP growth and population growth are positively correlated. This is the macro-view of the economy, which disregards interaction on the micro-level and seemingly predicts – in line with the available R&D-based growth literature – that higher population growth leads to higher productivity growth.

From micro-foundation, however, we have derived that both human capital and fertility are endogenous and *inversely* correlated via the quantity-quality trade-off. Along a balanced growth path with positive growth $w_t h_t$ is perpetually growing and fertility and the gross growth rate of human capital are constants. Inserting $\underline{n}$ and Δ_h from (5) and (7) into (17) provides the expression

$$\left(\frac{A_{t+1}}{A_t}\right)^{1-\phi} = A_E \frac{\gamma\tau}{\eta - \gamma}\left(\frac{\eta - \gamma}{(1+\beta+\eta)\tau}\right)^{1-\nu}.$$

Obviously, the most positive role that population growth could possibly play for TFP growth exists when there is no congestion in research, i.e. for $\nu = 0$. In this case the expression simplifies further and the balanced growth rate of TFP and – after inserting (5) into (15) – the population growth rate is obtained as in (18).

$$g_A = \left(\frac{\gamma A_E}{1+\beta+\eta}\right)^{1/(1-\phi)} - 1, \qquad g_L = \frac{\eta - \gamma}{(1+\beta+\eta)\tau} - 1. \tag{18}$$

Inspecting the growth rates shows that a higher weight of child quality in utility causes g_A to rise and g_L to fall. The opposite holds true for a decreasing weight of child quantity in utility. A proposition summarizes the main result of the paper.

PROPOSITION 1. *A higher weight on child quality γ or a lower weight on child quantity η implies a higher rate of TFP growth and a lower rate of population growth along the balanced growth path.*

This means that for two otherwise identical economies TFP growth is higher in the one in which parents put relatively less weight on child quantity, that is the one in which the fertility transition ends at a lower level of population growth. Congestion in research ($\nu > 0$) amplifies this result by reducing the role of n_t in TFP growth. More importantly, note that the result is independent from the size of n_t. In particular, it holds also when population growth $g_L = n_t - 1$ is negative. Ceteris paribus, declining population is good for TFP growth along the balanced growth path.

For an intuition of the result recall the definition of aggregate human capital $H_t = h_t L_t$. Without congestion a positive effect of declining population on productivity requires that a higher preference for child quality exerts a stronger effect on human capital endowment per person of the next generation than on the number of persons such that h_t grows more than L_t falls. This is exactly what our model-parents provide. Inserting (5) and (7) into $H_{t+1}/H_t = n_t \cdot (h_{t+1}/h_t)$ we obtain the growth rate of aggregate human capital along the BGP, which depends positively on γ (and negatively on η)

$$g_H = \frac{\eta - \gamma}{(1+\beta+\eta)\tau} \cdot \frac{\gamma \tau A_E}{\eta - \gamma} - 1 = \frac{\gamma A_E}{1+\beta+\eta} - 1 \qquad \Rightarrow \qquad \frac{\partial g_H}{\partial \gamma} = \frac{A_E}{1+\beta+\eta} > 0.$$

The mechanism behind the result originates from the interaction in the budget constraint (and not from specifications of the utility function). To see this clearly consider a unit increase of e_t in company with a unit reduction of n_t such that total voluntary child expenditure $n_t e_t$ remains constant. This one-to–one quantity-quality substitution is not neutral. It sets free income $\tau w_t h_t$ because less time is needed for child rearing so that more time can be supplied on the labor market. The additionally earned income can be spend on current and future consumption and on further child expenditure e_t implying that the negative effect from reduction of fertility is smaller than the positive effect on human capital such that $H_t = h_t n_t$ rises. Because the mechanism arises from the budget constraint, we are confident that the result holds also for more general forms of the utility function.

Turning towards the impact of time costs of children we see from (18) that a change of τ affects population growth but not productivity growth. Intuitively, rising costs of children lead to lower fertility and higher voluntary expenditure per child. For aggregate human capital $H_t = h_t L_t$ the negative effect through lower fertility and the positive effect via higher human capital growth per capita are exactly leveling each other such that $H_{t+1}/H_t = \gamma A_E/(1+\beta+\eta)$ is independent from τ and thus $\partial g_H/\partial \tau = 0$.

The mechanics behind the result originate again from the budget constraint, but this time log-utility and its feature of balancing income and substitution effects plays a role as well. Higher child costs lead to lower child demand n_t and lower available income $(1 - \tau n_t) w_t h_t$. With unchanged preferences income and substitution effect are balancing each other such that total expenditure

$n_t e_t$ remains constant. A utility function supporting a higher substitution effect would imply an overcompensating effect of human capital over fertility.

Besides manipulating the utility function, an effect of τ on TFP growth could also be motivated by congestion. If there is congestion in R&D, i.e. if $\nu > 0$ in $g_A = n_t^{1-\nu}(h_{t+1}/h_t)$, then the positive effect through rising human capital dominates the negative effect through falling fertility, an observation, which proves the following proposition.

Proposition 2. *If there is congestion in R&D (stepping on toes) then increasing time costs for rearing children leads to lower population growth and higher TFP growth along the balanced growth path.*

3.2. **Income Growth and Population Growth.** In order to examine the rest of the model, we evaluate (16) along the balanced growth path (i.e. for $\hat{g}_K = 0$) and substitute (h_{t+1}/h_t) from (17). This provides

$$\left(\frac{K_{t+1}}{K_t}\right) = \left(\frac{A_{t+1}}{A_t}\right)^{2-\phi}\left(\frac{L_{t+1}}{L_t}\right)^{\nu}. \tag{19}$$

Without congestion in R&D ($\nu = 0$) the model predicts that growth of physical capital along the balanced growth path correlates positively with TFP growth but not with population growth. For $\phi \to 1$ the model predicts that the capital stock grows at the rate of TFP growth. Note the difference to neoclassical growth theory, which predicts that the capital stock grows at the rate of TFP growth *plus* the rate of population growth. With human capital and R&D being endogenous, a positive association between capital growth and population growth emerges "only" when there is congestion in research.

Finally, substitute labor demand $H_t^Y = A_t/(\alpha\delta)$ into production (11) and take time-differences to get output growth $g_{Yt} = (1+g_{Kt})^{\alpha}(1+g_{At})^{(1-\alpha)(2-\phi)}(1+g_{Lt})^{\nu(1-\alpha)} - 1$. Insert this information into growth of output per worker $y_t = Y_t/L_t$, i.e. into $(1+g_{y_t}) = (1+g_{Y_t})/(1+g_{Lt})$. In order to evaluate income per capita growth along the balanced growth path insert g_A and g_K from (17) and (19) to arrive at (20).

$$1 + g_y = \left(\frac{h_{t+1}}{h_t}\right)^{\frac{2-\phi}{1-\phi}} n_t^{\frac{1-\nu}{1-\phi}}. \tag{20}$$

Superficial inspection suggests again a *seemingly* positive association between income growth g_y and population growth (fertility). With contrast to conventional R&D-based growth theory, our micro-foundation of fertility can again be utilized to reconcile the model's predictions with the empirical facts. Inserting $\underline{n}$ and Δ_h from (5) and (5) into (10) provides (21).

$$1 + g_y = \left(\frac{\gamma \tau A_e}{\eta - \gamma}\right)^{\frac{2-\phi}{1-\phi}} \cdot \left(\frac{\eta - \gamma}{(1+\beta+\eta)\tau}\right)^{\frac{1-\nu}{1-\phi}} . \tag{21}$$

Taking the derivatives with respect to γ and η provides a result analogously to Proposition 1.

Proposition 3. *A higher weight on child quality in utility or a lower weight on child quantity implies a higher rate of growth of income per capita and a lower rate of population growth along the balanced growth path.*

Furthermore, since $2 - \phi > 1 - \nu$:

Proposition 4. *Higher child-rearing costs τ imply a higher rate of growth of income per capita and a lower rate of population growth along the balanced growth path.*

It is instructive to compare R&D effort along the BGP with the earlier R&D-based growth models. Models of the first generation (Romer, 1990; Aghion and Howitt, 1992) predict constant TFP growth for a constant number of researchers. For this to be true the knife-edge assumption $\phi = 1$ has to hold. Models of the second generation (Jones, 1995, Segerstrom, 1998) predict based on $\phi < 1$ that constant TFP growth is realized for a constant population share of researchers and positive population growth, implying that constant economic growth requires a perpetually rising number of people employed in R&D and perpetually rising R&D expenditure. Ha and Howitt (2007) have argued that empirical evidence for the U.S. growth experience after 1950 supports models of the first generation. Models of the first generation, however, have the unpleasant features of being based on the knife-edge assumption $\phi = 1$ and of relying on a constant population. The present theory reconciles the earlier theories. It abandons the knife-edge assumption but preserves the empirical relevant associations between research effort and TFP.

Proposition 5. *Along the balanced growth path constant TFP growth is associated with a constant share of the population working in R&D and constant R&D expenditure share of GDP. These results hold true for $\phi < 1$ irrespective of whether the number of people employed in R&D is rising, constant, or declining. If the population stays constant, constant TFP growth implies a constant number of workers engaged in R&D.*

For a proof let the number of workers in goods production be denoted by L_t^Y. Begin with noting that the share of workers in goods production is given by $L_t^Y/L_t = (h_t L_t^Y)(h_t L_t)/ = H_t^Y/H_t$. Insert $H_t^Y = A_t/(\alpha\delta H_t)$ and the definitions of H_t and δ to get $L_t^Y/L_t = A_t^{1-\phi}/(\alpha\bar{\delta} h_t L_t^{1-\nu})$. Conclude from (17) that numerator and denominator of this expression are growing at equal rates at the steady-state. Thus L_t^Y/L_t stays constant implying a constant population share in R&D.

For the second part of the proof, R&D expenditure is given by $R_t = w_t H_t^A$ and its share of GDP by $R_t/Y_t = w_t H_t^A/Y_t$. Insert wages from (9) to get $R_t/Y_t = (1-\alpha)H_t^A/H_t^Y$, which is constant since H_t^Y/H_t and H_t^A/H_t are constant along the steady-state.

3.3. **Growth of Modern vs. Post Modern Economies.** Standard unified growth theory usually assumes that the fertility transition ends in the Modern Era when fertility arrives at the replacement rate or at a rate that supports positive population growth. Our model allows the fertility transition to end below replacement level, a phenomenon which we identified as the Post-Modern Era. Consider two otherwise identical economies, one in which preferences support non-negative population growth along the BGP (the modern economy) and one in which preferences imply negative population growth along the BGP (the post-modern economy). Applying Proposition 4 and 5 – which hold irrespective of the size of n – we arrive at the following conclusion.

Proposition 6. *Growth of income per capita is – ceteris paribus – higher for the post-modern economy than for the modern economy.*

The intuition for the result has been developed in the context of Propositions 1-5. Summarizing, a change of preferences which causes a child quantity-quality substitution frees extra parental time that is used to earn extra income of which a part is invested in education. As a consequence the positive impact of the preference change on education exceeds the negative impact on population size such that aggregate human capital $H = h \cdot L$ and TFP are growing at higher rates than before. If there is congestion in R&D a second positive effect shows up because there is less stepping-on-toes.

4. Adjustment Dynamics over the Very Long Run

The present theory of R&D-driven technological progress predicts a negative association of TFP growth and population growth, a result in line with the cross-country evidence for the second half of the 20th century. For the most part of human history, however, Kremer (1993) has impressively documented a positive association between population growth and TFP growth. These two facts can be explained in a unified theory of economic growth once we take into account that in pre-modern times market R&D contributed little to productivity growth and there was no (mass) education (Mokyr, 2002). Instead a Malthusian mechanism was operative such that TFP growth through learning-by-doing activities translated only very gradually to growth of income per capita (Galor, 2005).

In this section we investigate long-run adjustment dynamics towards balanced growth. Initially, in the pre-modern era, both non-negativity constraints, $w_t h_t - z \geq 0$ and $H_t = L_t h_t - A_t/(\alpha\delta) \geq 0$ are binding with equality implying that there is neither market R&D nor mass education. Growth is solely driven by learning-by-doing as in Kremer (1993) and in Galor (2005) during the Malthusian era. Our model does not entail a general prediction which of the two non-negativity constraints is relaxed first but in the following we propose a calibration where market R&D sets in first (with the first Industrial Revolution around 1760) and triggers later on, around 1860, the onset of (mass) education. Such a scenario seems not only to be in line with the historical evolution of England and Western Europe (Galor, 2005). It also allows us to investigate a *transitional period* in which R&D growth is fueled solely by population growth; i.e. by the mechanism that is assumed to drive growth *at the steady-state* according to the semi-endogenous R&D-based growth literature (Jones, 1995).

In order to see how our theory relates to the semi-endogenous R&D-based growth literature it is instructive to assume for a moment $A_E = 0$ such that schooling is unproductive and parents abstain from spending on education. Inserting h_{t+1}/h_t into (17) we get productivity growth

$$g_A = n_t^{\frac{1-\nu}{1-\phi}} - 1.$$

From this we indeed conclude that there is a unique positive association between productivity growth and population growth. The model has collapsed to an overlapping generations version of the well-known semi-endogenous growth model (Jones, 1995).

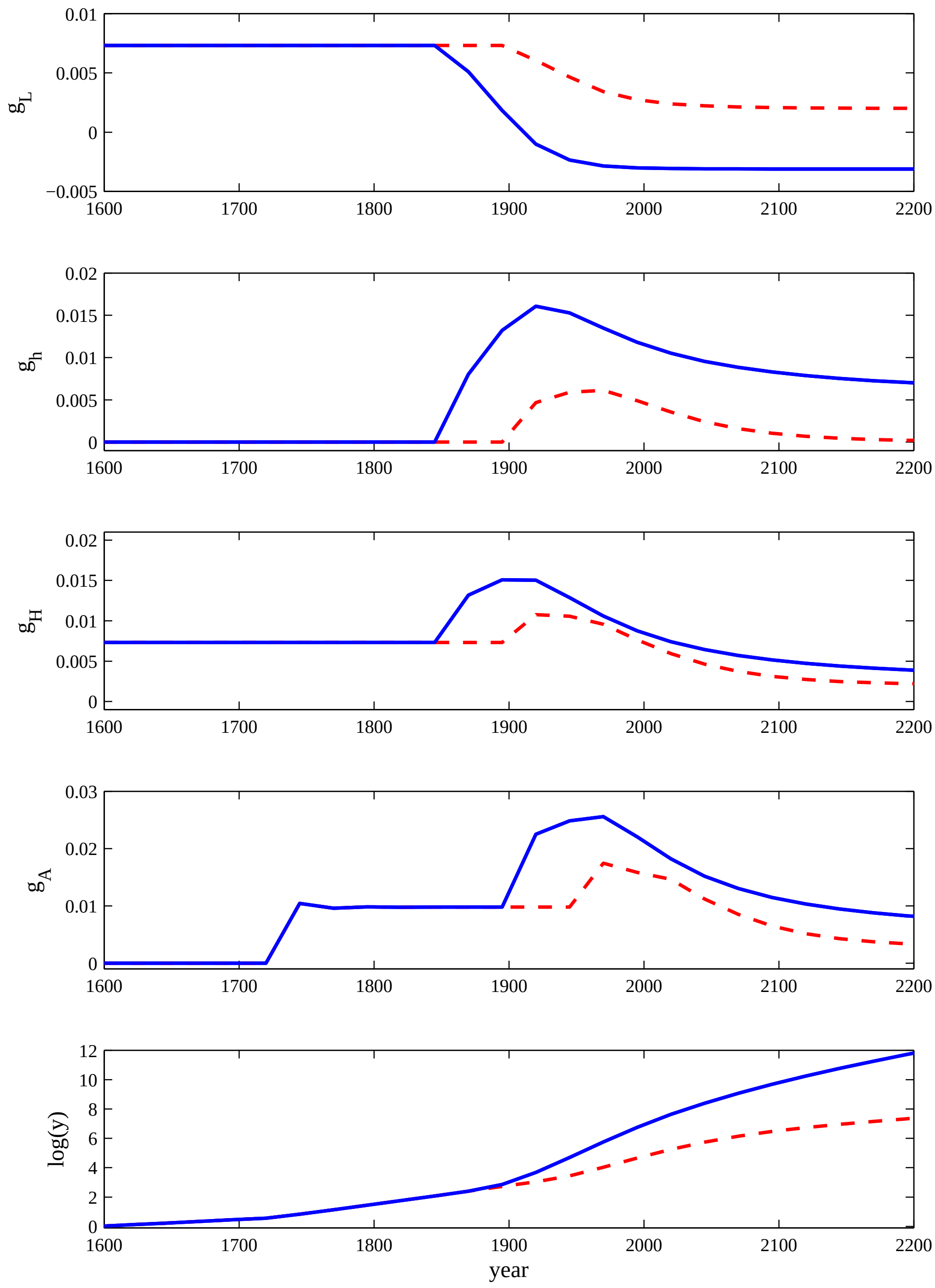
In a unified growth context, however, a period of R&D-based growth without education constitutes only a transitory phase and not a steady-state phenomenon. Eventually rising income triggers education, the education constraint becomes non-binding, and the demographic transition sets in. With rising human capital and declining fertility the economy converges towards the balanced growth path. While standard R&D-based growth theory would predict that economic growth declines with ongoing demographic transition because of declining population growth, the above steady-state analysis on the quality-quantity substitution lets us expect that a similar mechanism holds also along the adjustment path such that economic growth is expected to *increase* with the demographic transition. We verify the intuition with a numerical calibration of the model.

For the benchmark run we set the parameters β, γ, and η such that the model produces a savings rate of 0.2, a population growth rate of 0.2 before the demographic transition sets in, and one of -0.075 at the end of the demographic transition. Assuming that rearing a child takes away 10 percent of adult time ($\tau = 0.1$) this leads to the estimate $\beta = 0.29$, $\gamma = 0.04$, $\eta = 0.18$. After running the simulation we convert generational into annual data for better comparison with the actual historical time series. Assuming that a generation takes 25 years, the above setting implies a population growth rate of 0.92 percent before the onset of the transition and of -0.39 at the end of the transition. It also implies that two adults (a couple) have 2.4 surviving children before the onset of the demographic transition and 1.85 children after completion of the transition. The latter value matches the currently observed TFR in the UK and the TFR to which all countries are predicted to converge in the long-run according to the UN (2008) medium-variant projection.

We set $\alpha = 0.4$, $\phi = 0.5$, $\nu = 0.2$, and $A_E = 39$ such that along the balanced growth path income per capita grows at 1.5 percent annually, GDP grows at 1.1 percent and TFP grows at 0.7 percent. We assume a learning-by-doing function $g_B = \mu L^\lambda$ and set the remaining parameters λ, μ, $\bar{\delta}$, and $\bar{e}$ to roughly approximate the historical evolution of England. In particular we set these parameters (and the starting value L_0) such that R&D-based growth commences in the mid 18th century (patented innovations of the first Industrial Revolution), such that mass education and the demographic transition sets in in the mid 19th century (with the second Industrial Revolution), and such that a productivity slowdown sets in in the 1970s. This leads to the estimates $\lambda = \mu = 0.05$, $\bar{\delta} = 0.5$, and $\bar{e} = 0.03$.

Solid lines in Figure 2 show the implied adjustment dynamics. The model predicts correctly the onset of the fertility decline to occur in sync with the onset of mass education in the mid 19th century. It fails to predict, however, an increase of population growth during the 18th and early 19th century because we have not taken into account mortality. Accounting for mortality and its positive impact on net fertility would produce a hump-shaped transition path of population growth (See Herzer et al., 2011, Strulik and Weisdorf, 2011). The bottom panel in Figure 2 shows the evolution of income per capita (in logs). Income starts to increase somewhat with the onset of R&D-based growth and then really takes off with the onset of mass education. The quantity-quality substitution discussed

Figure 2: Long-Run Adjustment Dynamics



Solid lines: benchmark economy. Fertility of two adults stabilizes at 1.85 in line with the UN (2007) TFR projection. Dashed lines: alternative economy. Fertility of two adults stabilizes at 2.1 Both economies share the same parameter specification except $\gamma = 0.04$ for benchmark and $\gamma = 0.02$ for alternative economy (see text for details).

in Section 3 as a balanced growth phenomenon is also clearly visible as a phenomenon of adjustment dynamics: the growth rate of aggregate human capital (g_H, third panel) increases in line with the fertility decline because education (g_h, second panel) increases more strongly than fertility (g_L, first panel) decreases.

An increasingly well educated population leads with delay of one generation to a further increase of R&D-based growth in the late 18th and early 19th century (g_A, fourth panel). The model thus explains how a first Industrial Revolution brought forward by tinkerers initiated a second Industrial Revolution produced by formally trained scientists. In other words, it explains the transition from propositional knowledge towards prescriptive knowledge as the main driver of technological progress (Mokyr, 2005). In this sense, declining population growth was good for economic growth because it initiated a quality-quantity substitution of the workforce.

For the 21st century and beyond the model predicts human capital growth to decline towards its steady-state value and, as a consequence, growth of R&D and income per capita decline towards their steady-state levels as well. The model thus interprets the productivity slowdown as a phenomenon of adjustment dynamics, that is as overshooting behavior and adjustment towards "normal" from above.

Dashed lines in Figure 2 show adjustment dynamics for an alternative economy in which the transition ends at a fertility rate of 2.1 children per couple of adults, i.e. slightly above replacement rate. For that purpose we have reduced γ from 0.04 to 0.02 and kept all other parameters from the benchmark model. The higher preference for child quantity delays the onset of the fertility decline and mass education. More importantly, it leads also to less growth of human capital, initially as well as in the long-run. As a consequence the alternative economy produces less innovations (inferior g_A) and income per capita grows less steeply as in the benchmark economy. Our major results proved in the theoretical part as steady-state phenomena are thus also observed as phenomena of transitional dynamics: lower population growth is good for R&D-driven innovations and economic growth.

5. Conclusion

In this paper we have integrated into an R&D-based growth model an endogenous, microfounded evolution of population growth and human capital accumulation and have shown how this modifies some conclusions from earlier R&D-driven growth theory. While earlier models (in the spirit of Romer 1990 or Jones 1995) predicted that population growth is positively associated with economic growth, or even – in the Jones case – essential for having economic growth at all, our micro-founded theory predicts that population growth is negatively associated with productivity growth and income growth. It is therewith harder to falsify by the available data for the 20th century.

Since we have maintained all central elements about the firms' side from R&D-based growth theory it is clear that the new results originate from household behavior. The basic mechanism is generated by the interaction of child quality and quantity in the households' budget constraint and is observed independently from the specification of preferences, which makes us confident that our results are robust against a sophistication of the households' utility function.

Specifically, a substitution of child quantity n by child quality (i.e. expenditure on education) e that keeps total child expenditure $e \cdot n$ constant sets free parental time, which can be used to earn extra income. The additional income is partly spent on education such that overall child expenditure rises more strongly than child quantity falls. At the macro side of the economy this trade-off means that human capital per person h increases more strongly than the number of persons L falls such that total available human capital $h \cdot L$ increases. Given that human capital is the driving force in R&D this entails higher R&D output and higher R&D-based growth.

In a unified growth setting we have shown that this phenomenon is not only observable along the balanced growth path but also during the adjustment phase. The unified growth model has also produced novel insights about the timing of the onset of mass education and R&D-based growth. It is capable to explain how a first Industrial Revolution brought forward by tinkerers initiated a second Industrial Revolution produced by formally trained scientists. These details about the timing of long-run growth cannot be explained by the so far available growth theories because they neglect either R&D-based growth (unified growth theory) or the micro-foundation of fertility and human capital accumulation (conventional R&D-based growth theory).

Taking the quality-quantity trade-off into account allowed us to draw a much less grim conclusion about economic growth in the near future than suggested by the conventional R&D-based growth literature. The crucial ingredient that makes perpetual growth possible is not that there are constant returns in education for the individual. To see this note that along the balanced growth path the model predicts that the expenditure share of education is constant (the OLG equivalent of a constant share of life-time spent on education in the non-overlapping generations, Mincer-type approach to education). The model could thus be easily generalized towards decreasing returns at the individual level. The crucial ingredient enabling perpetual growth is the linear *intergenerational* transmission of human capital, i.e. the assumption that the current generation is capable to transport its knowledge times a multiplier larger than one to the next generation. While it is impossible to say whether such a process of knowledge transmission can be sustained forever, it is in any case easier conceivable than a perpetually growing population. Human capital is a metaphysical entity measured in value-units (compare, for example, the value of knowledge acquired by a university study of medical science now and 100 years ago) whereas population is a physical entity bounded by physical constraints, for example, space on earth.

Instead of venturing forth into the domain of speculation about the distant future of humanity we would like to emphasize that our model is a metaphor to explain economic growth in the past, the present, and the near future (say within the limit of the time horizon of a century). It is not a theory for economic growth in the very distant future. In the recent past, we observed high TFP growth in line with high growth of human capital and low and increasingly negative population growth, and we expect these trends to continue for a while. In this respect the main message delivered by the model is an optimistic one: the fact that fertility is below replacement level and population is declining is less threatening than suggested by conventional R&D-based growth theory.

In the very long-run it is likely that fertility below replacement and negative population growth run against physical and economic limits. This insight could have been another reason why the UN recently reconsidered its 2008 projections; assuming now adjustment to replacement level from below at the end of the 21. century. If population density becomes too thin we would expect indivisibilities to occur and technologies to become increasingly resistant against quantity-quality substitution. We would then expect that markets (or policies) react by generating a lower relative price of children. With ongoing adjustment of prices we would probably expect indeed convergence towards a stable population in the very long-run. Strulik and Weisdorf (2008) have developed a unified growth model of a two-final-goods economy, which endogenously produces convergence towards a stationary population in the very long-run and which predicts undershooting and negative population growth during the 21st century as a transitional phenomenon. Combining these ideas with the present work is a challenging task for the future.

References

Aghion, P. and P. Howitt, 1992, A model of growth through creative destruction, *Econometrica* 60, 323–702.

Aghion, P. and P. Howitt, 2009, *The Economics of Growth*, MIT Press, Cambridge, MA.

Ahituv, A., 2001, Be fruitful or multiply: On the interplay between fertility and economic development, *Journal of Population Economics* 14, 51-71.

Andreoni, J., 1989, Giving with impure altruism: Applications to charity and Ricardian equivalence, *Journal of Political Economy* 97, 1447-1458.

Baier, S.L., P.G. Dwyer Jr., and R. Tamura, 2006, How Important Are Capital and Total Factor Productivity for Economic Growth? *Economic Inquiry* 44, 23-49.

Barro, R.J., 2001, Human capital and growth, *American Economic Review* 91, 12–17.

Becker, G.S., 1960, An economic analysis of fertility, in: National Bureau of Economic Research (ed), *Demographic and Economic Change in Developed Countries*, Princeton University Press, Princeton, 209-231.

Becker, S.O. and Cinnirella, F. and Woessmann, L., 2010, The trade-off between fertility and education: evidence from before the demographic transition, *Journal of Economic Growth* 15, 177-204.

Bernanke, B.S. and Gürkaynak, R.S., 2001, Is growth exogenous? Taking Mankiw, Romer, and Weil seriously, *NBER macroeconomics annual* 16, 11-57.

Bongaarts, J., 2001, Fertility and reproductive preferences in post-transitional societies, *Population and Development Review* 27, 260-281.

Bongaarts, J., 1999, Fertility decline in the developed world: Where will it end?, *American Economic Review* 89, 256–260.

Bottazi, L. and G. Peri, 2003, Innovation, demand, and knowledge spillovers: Evidence form European patent data, *European Economic Review* 47, 687-710.

Brander, J.A., and S. Dowrick, 1994, The Role of Fertility and Population in Economic Growth, *Journal of Population Economics* 7, 1-25.

Caldwell, J.C. and Schindlmayr, T., 2003, Explanations of the fertility crisis in modern societies: A search for commonalities, *Population Studies* 57, 241–263.

Ciccone, A. and Papaioannou, E., 2009, Human capital, the structure of production, and growth, *The Review of Economics and Statistics* 91, 66–82.

Comin, D., Easterly, W., and Gong, E., 2010, Was the wealth of nations determined in 1000 BC?, *American Economic Journal: Macroeconomics* 2, 65-97.

Connolly, M. and Peretto, P.F., 2003, Industry and the family: two engines of growth, *Journal of Economic Growth* 8, 115–148.

Dalgaard, C.-J. and Kreiner, C.T.,, 2001, Is declining productivity inevitable?, *Journal of Economic Growth* 6, 187-203.

Dalgaard, C.-J. and Jensen, M.K., 2009, Life cycle savings, bequest and a diminishing impact of scale on growth, *Journal of Economic Dynamics and Control* 33, 1639-47.

Dinopoulos, E. and P. Thompson, 1998, Schumpeterian growth without scale effects, *Journal of Economic Growth* 3, 313-335.

Funke, M. Strulik, H., 2000, On endogenous growth with physical capital, human capital, and product variety', *European Economic Review* 44, 491-515.

Galor, O., 2005, From stagnation to growth: unified growth theory, in: P. Aghion and S. Durlauf, *Handbook of Economic Growth* Vol 1A, Elsevier, Amsterdam.

Galor, O., 2011, *Unified Growth Theory*, Princeton University Press, Princeton.

Goldstein, J. and Lutz, W. and Testa, M.R., 2003, The emergence of sub-replacement family size ideals in Europe, *Population Research and Policy Review* 22, 479–496

Grossmann, V., 2010, Entrepreneurial innovation and economic growth, *Journal of Macroeconomics*, forthcoming.

Growiec, J., 2006, Fertility choice and semi-endogenous growth: where Becker meets Jones, *Topics in Macroeconomics* 6, Article 10.

Ha, J. and P. Howitt, 2006, Accounting for trends in productivity and R&D, *Journal of Money, Credit, and Banking* 39, 733-774.

Hanushek, E.A., 1992, The trade-off between child quantity and quality, *Journal of Political Economy* 100, 84-117.

Herzer, D., Vollmer, S., and Strulik, H., 2010, The long-run determinants of fertility: One century of demographic change 1900-1999, Discussion Paper, University of Hannover.

Howitt, P. 1999, Steady endogenous growth with population and R&D inputs growing, *Journal of Political Economy* 107, 715-30.

Jaffe, A. M. Tratjenberg, and R. Henderson, 1993, Geographic localization of knowledge spillovers as evidenced by patent citations *Quarterly Journal of Economics* 63, 577-598.

Jones, C.I., 1995, R&D-based models of economic growth, *Journal of Political Economy* 103, 759-784.

Jones, C.I., 1999, Growth: with or without scale effects? ,*American Economic Review Papers and Proceedings* 89, 139-44.

Jones, C.I., 2001, Was an industrial revolution inevitable? Economic growth over the very long run, *Advances in Macroeconomics* 1, 1-43.

Kelley, A.C. and Schmidt, R., 1995, Aggregate population and economic growth correlations: The role of the components of demographic change", *Demography* 32, 543-55.

Keller, W., 2002, Geographic localization of international technology diffusion, *American Economic Review* 92, 120-142.

Kortum, S, 1997, Research, patenting, and technological change, *Econometrica* 65, 1389-1419.

Kremer, M., 1993, Population growth and technological change: one million B.C. to 1990, *Quarterly Journal of Economics* 108, 681-716.

Krueger, A.B. and Lindahl, M., 2001, Education for growth: why and for whom?, *Journal of Economic Literature*, 39, 1101-1136.

Lee, R., and Mason, A., 2010, Fertility, human capital, and economic growth over the demographic transition, *European Journal of Population* 26, 159-182.

Li, C.-W., Endogenous vs. semi-endogenous growth in a two R&D-sector model", *Economic Journal*, C109-C122.

Lucas, R. E., 1988, On the mechanics of economic development, *Journal of Monetary Economics* 22, 3-42.

Mankiw, G., D. Romer and D. Weil, 1992, A contribution to the empirics of economic growth. *Quarterly Journal of Economics* 107,407-37.

Mokyr, J., 2002, *The Gifts of Athena*, Princeton University Press, Princeton.

Mokyr, J. 2005, Long-term economic growth and the history of technology, in: Aghion, P. and S.N. Durlauf (eds.) *Handbook of Economic Growth*, Elsevier, Amsterdam, Chapter 17.

Peretto, P.F., 1998, Technological change and population growth, *Journal of Economic Growth* 3, 283-311.

Preston, S.H. and Hartnett, C.S., 2008, The future of American fertility, NBER Working Paper 14498.

Romer, P.M., 1990, Endogenous technological change, *Journal of Political Economy* 98, S71-S102.

Rosenzweig, M. 1990, Population growth and human capital investments: theory and evidence, *Journal of Political Economy* 98, S38-S70.

Rosenzweig, M., and K.I. Wolpin, 1980, Testing the quantity-quality fertility model: the use of twins as natural experiment, *Econometrica* 48, 227-240.

Segerstrom, P., 1998, Endogenous growth without scale effects, *American Economic Review* 88, 1290-1310.

Segerstrom, P., 2000, The Long-Run Growth Effects of R&D Subsidies", *Journal of Economic Growth* 5, 277-305.

Strulik, H., 2005, The Role of Human Capital and Population Growth in R&D-Based Models of Economic Growth, *Review of International Economics* 13, 129-145.

Strulik, H. and J. Weisdorf, 2008, Population, food, and knowledge: A simple unified growth model, *Journal of Economic Growth* 13, 169-194.

Strulik, H. and S. Vollmer, 2010, The fertility transition around the World 1950-2005, Working Paper, Harvard University.

UN, 2008, *United Nations World Population Prospects: 2008 revision* United Nations Department of Economic and Social Affairs, Population Division. New York.

UN, 2011, *United Nations World Population Prospects: 2010 revision* United Nations Department of Economic and Social Affairs, Population Division. New York. [Update 28. June 2011, http://esa.un.org/unpd/wpp/].

U.S. National Center for Health Statistics. 2010, National Vital Statistics Reports Volume 58 (16), Births: Preliminary Data for 2008, U.S. Department of Health and Human Services.

van de Kaa, D.J., 2001, Postmodern fertility preferences: From changing value orientation to new behavior, *Population and Development Review* 27, 290-331.

Weil, D., 2005, *Economic Growth*, accompanying data set, Pearson, Boston.

Young, A., 1998, Growth Without Scale Effects, *Journal of Political Economy* 106, 41-63.